

## Continuity

## And Differentiability

### SECTION – A

Questions 1 to 10 carry 1 mark each.

- The function  $f(x) = [x]$ , where  $[x]$  denotes the greatest integer function, is continuous at:  
(a) 4      (b) 1.5      (c) 1      (d) -2
- If  $x = at^2$ ,  $y = 2at$ , then  $\frac{d^2y}{dx^2}$  is  
(a)  $\frac{1}{t}$       (b)  $-\frac{1}{t^2}$       (c)  $at^2$       (d)  $-\frac{1}{2at^3}$
- If  $y = \sin^{-1}\left(\frac{3x}{2} - \frac{x^3}{2}\right)$ , then  $\frac{dy}{dx}$  is  
(a)  $\frac{3}{\sqrt{4-x^2}}$       (b)  $\frac{-3}{\sqrt{4-x^2}}$       (c)  $\frac{1}{\sqrt{4-x^2}}$       (d)  $\frac{-1}{\sqrt{4-x^2}}$
- If  $y = Ae^{5x} + Be^{-5x}$  then  $\frac{d^2y}{dx^2}$  is equal to  
(a)  $25y$       (b)  $5y$       (c)  $-25y$       (d)  $10y$
- Derivative of  $\sin x$  with respect to  $\log x$ , is  
(a)  $\frac{x}{\cos x}$       (b)  $\frac{\cos x}{x}$       (c)  $x \cos x$       (d)  $x^2 \cos x$
- The function  $f(x) = x|x|$  is  
(a) continuous and differentiable at  $x = 0$ .  
(b) continuous but not differentiable at  $x = 0$ .  
(c) differentiable but not continuous at  $x = 0$ .  
(d) neither differentiable nor continuous at  $x = 0$ .
- A function  $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ 2k, & x = 0 \end{cases}$  is continuous at  $x = 0$  for  
(a)  $k = 1$       (b)  $k = 2$       (c)  $k = \frac{1}{2}$       (d)  $k = \frac{3}{2}$

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8. If  $y = \tan^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ , then  $\frac{dy}{dx}$  is equal to

- (a)  $\frac{1}{1+x^4}$       (b)  $\frac{-2x}{1+x^4}$       (c)  $\frac{-1}{1+x^4}$       (d)  $\frac{x^2}{1+x^4}$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).  
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).  
(c) Assertion (A) is true but reason (R) is false.  
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** Every differentiable function is continuous but converse is not true.

**Reason (R):** Function  $f(x) = |x|$  is continuous.

10. **Assertion (A):**  $f(x) = |x - 3|$  is continuous at  $x = 0$ .

**Reason (R):**  $f(x) = |x - 3|$  is differentiable at  $x = 0$ .

## SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find all points of discontinuity of  $f$ , where  $f$  is defined by  $f(x) = \begin{cases} 2x+3, & \text{if } x \leq 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$ .

12. Find the values of  $k$  so that the function  $f(x) = \begin{cases} kx+1, & \text{if } x \leq 5 \\ 3x-5, & \text{if } x > 5 \end{cases}$  is continuous at point  $x = 5$ .

13. Differentiate  $\sin(\tan^{-1} e^{-x})$  with respect to  $x$ .

14. Find  $\frac{dy}{dx}$  if  $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ ,  $0 < x < 1$

## SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find  $\frac{dy}{dx}$  if  $x = a(\theta + \sin \theta)$ ,  $y = a(1 - \cos \theta)$ .

16. Prove that the function  $f$  given by  $f(x) = |x - 1|$ ,  $x \in \mathbb{R}$  is not differentiable at  $x = 1$ .

17. If  $y = 3e^{2x} + 2e^{3x}$ , prove that  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

## SECTION – D

Questions 18 carry 5 marks.

18. Differentiate  $(\log x)^x + x^{\log x}$  with respect to  $x$ .

## SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. A potter made a mud vessel, where the shape of the pot is based on  $f(x) = |x - 3| + |x - 2|$ , where  $f(x)$  represents the height of the pot.

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- (a) When  $x > 4$  What will be the height in terms of  $x$ ? (1)
- (b) What is  $\frac{dy}{dx}$  at  $x = 3$ ? (1)
- (c) When the  $x$  value lies between (2, 3) then the function is (1)
- (d) If the potter is trying to make a pot using the function  $f(x) = [x]$ , will he get a pot or not? Why? (1)

20. Sumit has a doubt in the continuity and differentiability problem, but due to COVID-19 he is unable to meet with his teachers or friends. So he decided to ask his doubt with his friends Sunita and Vikram with the help of video call. Sunita said that the given function is continuous for all the real value of  $x$  while Vikram said that the function is continuous for all the real value of  $x$  except at  $x = 3$ .

The given function is  $f(x) = \frac{x^2 - 9}{x - 3}$

Based on the above information, answer the following questions:

- (a) Whose answer is correct? (1)
- (b) Find the derivative of the given function with respect to  $x$ . (1)
- (c) Find the value of  $f'(3)$ . (1)
- (d) Find the second differentiation of the given function with respect to  $x$ . (1)

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